Summary Report

SONOTRON PROTOCOL I

A Multicenter Short Term Single Administration Double Blind Randomized Placebo Controlled Study of the Efficacy and Safety of Sonotron in the Treatment of Pain in Human Subjects with Osteoarthritis of the knee.

Prepared by

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Abstract: A single administration study of the human use version of the Sonotron device was conducted to evaluate its effect on the knee joint in subjects with osteoarthritis under FDA IDE part 812.2.b provisions. The study was conducted at 5 regional centers on a population of 98 human subjects. Data were analyzed by Chi-Squared analysis on eleven non-parametric measures to compare the relative responses of the randomly assigned control and treated subpopulations. Two datasets showed a high probability that the subjects' assessment of pain one week after administration was reduced in the treated relative to the untreated control subpopulation. These measures were knee joint pain on passive motion ($p=0.001$) and knee joint pain at rest ($p=0.003$).

Certain other data suggested a trend of improvement one week after administration in the treated relative to the untreated group, but with lower probability. These measures were global evaluations by the investigator ($p=0.026$) and knee joint pain on weight bearing ($p=0.068$).

None of 98 subjects in the study reported adverse reactions to administration of the Sonotron which the investigators deemed significant or long lasting. Those subjects reporting any sensation (twelve in the treated group; 3 in the control group) indicated that in all cases the reaction was transitory and none indicated any perceived effects at the one week recheck other than diminished pain.

Background

Sonotron is a non-invasive device which employs modulated radio-frequency (RF) energy in the form of a visible and audible discharge beam eminating from a discharge electrode. The device is operated from 110/120 VAC 60 Hz power. The source of energy is an RF generator continuously adjustable from 0-10 watts with a carrier frequency approximately 430,000 Hz. It utilizes electromagnetic energy modulated at a lower frequency of 3,000-5,000 Hz. The discharge electrode is covered by a heat resistant glass tube and recessed within a plastic housing to assure that the electrode remains at the constant preset distance from the skin. It is to be employed as a therapeutic device which utilizes radiofrequency energy transmitted through the air directly to the skin for relief of pain associated with arthritis.

Tests have been conducted by the Biomedical Engineering staff of the Instrumentation Systems Center in the College of Engineering of the University of Wisconsin at Madison to quantify and classify the output signals of the Sonotron device. The results of these tests are included in Appendix II.

Output signals were found to resemble the frequency spectra of electrosurgical units, but with approximately 1/60th the current and power equivalent of power to a night light or a single christmas tree lamp. The amount of power simulations indicate might be dissipated within tissue was even smaller, similar in magnitude to the power of a pocket calculator. The scientific literature indicates that the level should be at or below the threshold of perception and the power level small enough to preclude a burn hazard even in the event of an inadverent ground contact. Based on these data, the Sonotron device should be regarded as a non-significant risk device. This conclusion is based upon comparison of the output of the device to other electromedical devices presently in use with similar frequency content and the literature reports on the effects of high frequency currents on human tissue.

The Sonotron device represents a new and unique therapeutic modality. Preliminary evidence in animal studies suggests that Sonotron therapy may be beneficial in rheumatic diseases. The Sonotron device has been used as a treatment modality in rats with adjuvent induced arthritis. Sonotron treated rats were compared with placebo treated animals. As compared to the placebo treated animals there was no evidence of either local or systemic tissue injury as a result of Sonotron treatment. The Sonotron treated animals showed less soft tissue swelling and joint destruction than did the placebo animals.

Given this background, the present study will evaluate the efficacy and the safety of single dose administration of Sonotron as compared to placebo in patients with osteoarthritis of the knee. The study design will permit recognition of the timing of onset of acute pain relief. Patient follow-up will allow assessment for the duration of any benefit observed, assessment of more delayed onset of benefit and assessment for local toxicity.

Objectives

1. To determine the efficacy of a single administration of Sonotron as compared to placebo in relief of pain secondary to osteoarthritis of the knee.

2. To determine the rapidity of onset of pain relief and the duration of pain relief up to one week following a single administration of Sonotron.

3. To compare the safety and tolerance of Sonotron and placebo following a single administration.

Patient Selection

A. Criteria for admission to the study.

Patients must fulfill all of the following criteria for admission to the study.

1. Ambulatory outpatients of age greater than 21.

2. A clinical diagnosis of osteoarthritis of the knee.

3. Roentgenographic evidence of osteoarthritis of the knee to be studied, obtained within six months of study entry.

4. Pain in the knee to be studied. Pain should be present on passive motion or weight bearing. A pain score representing the sum of the scores for pain at rest, on passive motion and on weight bearing must be equal to or greater than 9,(see Appendix I, Form 3).

5. Pain should be sustained despite concomitant non-steroidal anti-inflammatory or analgesic medication or a flare of pain following withdrawal of medication sufficient to fulfill criteria 4 must be documented prior to study entry.

6. Symptomatic osteoarthritis should have been present for at least three months.

7. Willingness to participate as evidenced by a signed informed consent.

B. Exclusions:

1. Circulatory impairment of the lower extremity or previous major arterial vascular surgery.

2. A history of gout, calcium pyrophosphate arthritis or other inflammatory arthritis.

3. Chondrocalcinosis of the knee by roentgenogram.

4. Infection, tumor or significant skin rash in or overlying the knee or surrounding soft tissue.

5. Impaired sensory function in the lower extremity.

6. Any systemic disorder which might interfere with or impair evaluation.

7. Non-ambulatory patient.

8. Prosthetic joint replacement or placement of any metallic device in or around the knee.

9. Intraarticular corticosteroids within the preceding three months or parenteral corticosteroids within the preceding month.

10. Treatment with oral or injectable gold, any other remission inducing or immunosuppressive drug during the past six months.

11. Patients participating in a trial of another experimental treatment.

12. Patients unable or unwilling to comply with all study requirements.

13. Grade II or greater instability of the knee in any plane.

14. Patients with a cardiac pacemaker.

Study Design

A. Twenty patients will be studied at each center. Patients who meet the entrance criteria will be assigned to receive either Sonotron or placebo treatment by a pre-determined randomized assignment schedule. To maintain the double blind nature of the study Sonotron treatment and placebo treatment will be administered utilizing identical appearing apparatus which will emit similar light and produce similar sound.

At each treatment visit treatment will be administered to the anterior and posterior surface of the joint. Treatment will be applied for 45 seconds at three consecutive 15 second bursts. The total application to the joint will be 90 seconds.

B. Patient Evaluation

1. Assessment At Study Entry:

Patients will be examined at the time of selection for the study. The following forms will be completed, (Appendix I):

a. History, physical exam (Form 1).

b. Entry criteria (Form 2).

c. Joint evaluation form (Form 3).

d. Assessment of Acute Response (Form 4).

e. Laboratory result (Form 5).

f. Adverse experiences (Form 6).

All narcotic analgesics will be discontinued at least 48 hrs. prior to study entry. Patients who have been taking a constant daily dose of a non-steroidal anti-

inflammatory drug or non-narcotic analgesic, and who fulfill study entry criteria including pain criteria may be permitted to continue their medications at the same daily dose during the study. Anti-inflammatory or analgesic medication which will not be maintained throughout the study or do not meet study entry criteria should be discontinued one week prior to study entry.

Baseline bloods will be obtained (Form 5, CBC, urinalysis, creatinine, blood urea nitrogen(BUN), uric acid, SGOT, alkaline phosphatase, total bilirubin. lactic acid dehydrogenase, calcium, phosphorous, potassium).

2. Assessment of acute response to Sonotron (Appendix 1, Form 4).

Patients will be evaluated for pain at rest, pain on passive motion, pain on weight bearing and tenderness immediately prior to treatment and at 30, 60, 90, 120 and 180 minutes after treatment. In addition, the skin and soft tissue will be examined and any adverse effects are recorded at each time interval. Global evaluation of benefit and response will be recorded by both the patient and the physician.

3. Follow-up Assessment

Follow-up visit evaluation will be conducted seven days after treatment. Evaluation for response and adverse effects will be completed and laboratory tests repeated, (Form 5).

General Considerations

All evaluations on any patient should be carried out by the same evaluator and as close to possible at the same time of day. All case report forms should be filled out completely. Triplicate forms will be provided. One copy is to be retained by the investigator. The other two will be collected by the study monitor.

Adverse reactions must be recorded. Any serious adverse reaction should be reported immediately by telephone to

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(608) 263-3892

Study Termination and Protocol Violations.

Patients may terminate the study at any time and for any reason. All study forms prior to termination must be completed and retained.

The investigator should make every effort to retain the patient in the study when feasible and ethical. Reasons for termination should be documented.

All protocol violations should be recorded and reasons documented when known.

Data Analysis

All data will be recorded on standardized case report forms provided to the investigator. Case report forms should be completed promptly at the time of the visit.

Data will be analyzed for each investigator separately and for all investigators if more than one center is involved. Incomplete cases or cases with significant protocol violations will be excluded from the efficiency analysis.

Efficacy parameters will be analyzed individually and subject to statistical comparison.

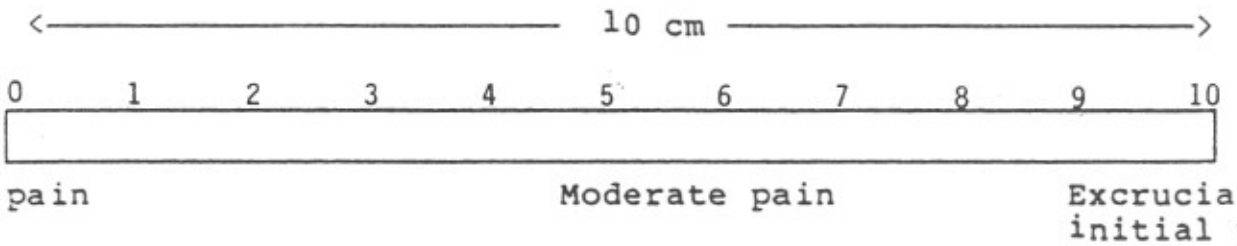
Daily Patient Log

Patients will be requested to complete a daily log recording use of concomitant medication and pain severity, (Appendix I, Form 7). The data will not be used for statistical analysis, but will provide information on duration of any beneficial effect observed.

Instructions for Evaluations

A. Overall Pain Scale

Pain will be assessed for the study joint and scored by a visual analog pain scale. The patient will indicate by pointing to the region of the scale which represents the current level of pain.



The following pain measures will be assessed:

Pain at rest

Pain on passive motion of the affected joint

Pain on weight bearing

B. Tenderness

Tenderness in the involved joint will be assessed according to the following scale.

0 = no response or quivering upon examination

1 = positive response or quivering upon examination

2 = spontaneous response upon examination

3 = withdrawal from contact by patient upon examination

C. Limitation of Motion

The investigator will move the involved joint through the joint's normal range of motion. A goniometer will be used to measure the range of motion. The limits to passive motion will be defined by patient's indication of moderate pain or when the investigator feels moderate resistance. Baseline range will be established upon entry into the study for each subject and remeasured at each visit.

Criteria for improvement of range of motion will be as follows:

Failure	=	less than 30% increase in range of motion
Minimal Response	=	between 30% and 50% increase in range of motion
Moderate Response	=	between 50% and 75% increase in range of motion
Excellent Response	=	greater than 75% increase in range of motion

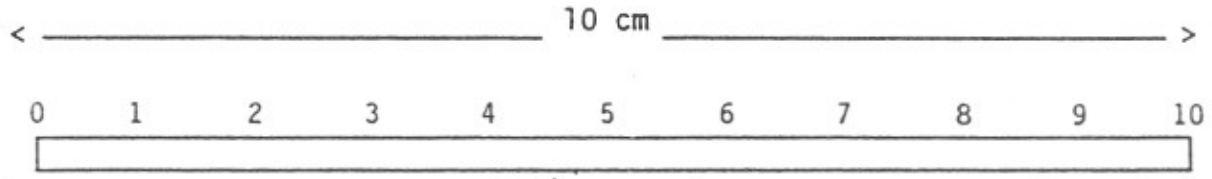
D. Fifty Foot Walking Time

The time required for the patient to walk a marked distance of 50 feet from a standing start will be recorded to the nearest 0.1 seconds. A stop watch should be utilized. The patient should not use a walking aid other than a cane if one is routinely used by the patient. If a cane is used, it should be used at every evaluation of walking time.

E. Global Evaluation of Patient Response by Patient and Investigator

This represents the overall evaluation by the investigator and separately by the patient of the clinical status of the affected

joint. The investigator should take care and be certain that the response refers solely to the joint under evaluation without taking account of disease activity in other joints. Initial evaluation will be recorded on a visual analog scale which is 10 cm in length where a marking at the 0 end of the scale will represent no abbreviated range of motion, a marking at the 10 cm will represent extreme limitation in range of motion and excruciating pain. The investigator should complete his own evaluation before requesting evaluation by the patient. The patient should not be shown the investigator's evaluation.



No pain
No limitation ROM
No functional abnormality

Moderate pain
Moderate decreased ROM
Moderate functional ROM

Excruciating pain
Severe decreased ROM & functional abnormality

At the follow-up visit, the investigator and the patient will be asked to indicate their judgement of the overall response to treatment. In this regard, they will take into consideration both the efficacy and any side effects encountered. Both investigator and patient will separately record their global performance evaluation of response from first to last assessment. Response following treatment will be according to the following semi-quantitative scale:

Failure	=	less than 30% improvement
Minimal response	=	between 30% and 50% improvement
Moderate response	=	between 50% and 75% improvement
Excellent response	=	greater than 75% improvement

Joint Swelling

The investigator will record the presence or absence of joint swelling in the effected joint according to the following scale:

0 = no swelling

1 = minimal swelling, (positive bulge sign only or rarely detectable swelling)

2 = minimal but definite effusion with no loss of distinct anatomic markings

3 = moderate effusion with loss of distinct anatomic markings

4 = large effusion with loss of all anatomic markings-severe bulge

Chi-Squared Analysis Results

The accompanying pages detail the results of the statistical study done on the Sonotron results.

Each block of data has five numbers in it.

The number in the upper left of each block is the Probability of the treated group and the control group being a single population (indistinct). A probability of <.050 would indicate that the treatment had some measurable affect. Fields where this probability exist have been highlighted.

The remaining four numbers are arranged in a matrix like so:

	Treated	Untreated
Improved	A	B
Deproved or No Change	C	D

So that A is the number of treated patients that showed improvement; and C is those treated patients who did not improve, or got worse; etc.

The mathematics of the Chi-Squared Technique are as follows:

$$Chi^2 = [(A-B)^2/(A+B)] + [(C-D)^2/(C+D)]$$

This number is looked up on the chart (see attached Appendix) and the probability determined from that position. The probability as reported is 1 minus the probability in the chart.

You can easily see that the most noticable distinction comes from the Joint Pain - Passive Range of Motion data. All other data sets are much more sporadic.

Results of Bio-Statistician's Chi-Squared Analysis

Locat.	Range of Motion: Flexion (Degrees) 30		60		120		180		1 wk	
Overall	0.946		0.862		0.108		0.846		0.913	
	5	15	8	9	13	7	7	8	11	11
	43	45	40	41	35	43	41	42	37	39
MSN-VA	NA		NA		NA		NA		NA	
	0	0	0	0	0	0	0	0	0	0
	9	10	9	10	9	10	9	10	9	10
NEBR	0.264		0.606		0.639		0.329		0.606	
	1	3	2	3	4	3	2	4	2	3
	9	7	8	7	6	7	8	6	8	7
RAC-NYC	NA		NA		NA		NA		0.893	
	0	0	0	0	0	0	0	0	2	2
	9	8	9	8	9	8	9	8	7	6
STVH	NA		NA		NA		NA		0.671	
	0	0	0	0	0	0	0	0	1	2
	8	10	8	10	8	10	8	10	7	8
MIAMI	0.346		1.001		0.041		0.673		0.206	
	4	2	6	6	9	4	5	4	6	3
	8	10	6	6	3	8	7	8	6	9

Results of Bio-Statistician's Chi-Squared Analysis

Range of Motion: Extension (Degrees)

	30		60		120		180		1 wk	
Overall	0.325		0.68		0.085		0.305		0.262	
	0	1	2	3	0	3	1	0	2	5
	48	49	46	47	48	47	47	50	46	45
MSN-VA	NA		NA		NA		NA		NA	
	0	0	0	0	0	0	0	0	0	0
	9	10	9	10	9	10	9	10	9	10
NEBR	NA		0.305		0.136		0.305		0.606	
	0	0	0	1	0	2	0	1	2	3
	10	10	10	9	10	8	10	9	8	7
RAC-NYC	NA		NA		NA		NA		NA	
	0	0	0	0	0	0	0	0	0	0
	9	8	9	8	9	8	9	8	9	8
STVH	NA		NA		NA		NA		NA	
	0	0	0	0	0	0	0	0	0	0
	8	10	8	10	8	10	8	10	8	10
MIAMI	0.307		1		0.307		NA		0.14	
	0	1	2	2	0	1	0	0	0	2
	12	11	10	10	12	11	12	12	12	10

Results of Bio-Statistician's Chi-Squared Analysis

Joint Pain: Rest: (0 - 10 Scale)

	30		60		120		180		1 wk	
Overall	0.903		0.888		0.936		0.883		0.003	
	13	13	16	16	15	16	17	17	23	10
	35	37	32	34	33	34	31	33	25	40
MSN-VA	0.701		0.764		0.764		0.764		0.018	
	2	3	3	4	3	4	3	4	4	0
	7	7	6	6	6	6	6	6	5	10
NEBR	NA		NA		NA		NA		1	
	0	0	0	0	0	0	0	0	2	2
	10	10	10	10	10	10	10	10	8	8
RAC-NYC	0.49		0.232		0.772		0.772		0.229	
	2	3	2	4	4	3	4	3	6	3
	7	5	7	4	5	5	5	5	3	5
STVH	0.737		0.671		0.502		0.671		0.41	
	3	3	4	4	2	4	4	4	3	2
	5	7	4	6	6	6	4	6	5	8
MIAMI	0.408		0.219		0.682		1		0.041	
	6	4	7	4	6	5	6	6	8	3
	6	8	5	8	6	7	6	6	4	9

Results of Bio-Statistician's Chi-Squared Analysis

Joint Pain: Passive ROM: (0 - 10 Scale)

	30		60		120		180		1 wk	
Overall	0.147		0.014		0.044		0.016		0.001	
	18	12	24	13	26	17	28	17	29	13
	30	38	24	37	22	33	20	33	19	37
MSN-VA	0.906		0.701		0.906		0.906		0.466	
	2	2	2	3	2	2	2	2	2	1
	7	8	7	7	7	8	7	8	7	8
NEBR	NA		NA		0.305		0.136		0.606	
	0	0	0	0	0	1	0	2	3	2
	10	10	10	10	10	9	10	8	7	8
RAC-NYC	0.893		0.064		0.024		0.007		0.002	
	2	2	5	1	6	1	7	1	8	1
	7	6	4	7	3	7	2	7	1	7
STVH	0.015		0.019		0.094		0.019		0.058	
	7	3	8	5	7	5	8	5	6	3
	1	7	0	5	1	5	0	5	2	7
MIAMI	0.414		0.041		0.132		0.059		0.083	
	7	5	9	4	11	8	11	7	10	6
	5	7	3	8	1	4	1	5	2	6

Results of Bio-Statistician's Chi-Squared Analysis

Joint Pain: Weight Bearing: (0 - 10)

	30		60		120		180		1 wk	
Overall	0.434		0.427		0.16		0.156		0.068	
	19	16	24	21	26	20	28	22	29	21
	29	34	24	29	22	30	20	28	19	20
MSN-VA	0.906		0.51		0.466		0.466		0.466	
	2	2	3	2	2	1	2	1	2	1
	7	8	6	8	7	9	7	9	7	9
NEBR	NA		1		1		0.531		0.653	
	0	0	1	1	1	1	1	2	5	4
	10	10	9	9	9	9	9	8	5	6
RAC-NYC	0.232		0.819		0.486		0.486		0.858	
	2	4	4	4	6	4	6	4	6	5
	7	4	5	4	3	4	3	4	3	3
STVH	0.058		0.094		0.28		0.019		0.387	
	6	3	7	5	6	5	8	5	4	3
	2	7	1	5	2	5	0	5	4	7
MIAMI	0.386		1		0.273		0.537		0.028	
	9	7	9	9	11	9	11	10	12	8
	3	5	3	3	1	3	1	2	0	4

Results of Bio-Statistician's Chi-Squared Analysis

Joint Swelling: (0 - 4 Scale)

	30		60		120		180		1 wk	
Overall	0.967		0.429		0.959		0.654		0.084	
	2	2	2	4	3	3	4	3	11	5
	46	48	46	46	45	47	44	47	37	45
MSN-VA	NA		NA		NA		NA		NA	
	0	0	0	0	0	0	0	0	0	0
	9	10	9	10	9	10	9	10	9	10
NEBR	NA		NA		NA		NA		0.305	
	0	0	0	0	0	0	0	0	1	0
	10	10	10	10	10	10	10	10	9	10
RAC-NYC	0.274		0.274		0.274		0.274		0.149	
	0	1	0	1	0	1	0	1	4	1
	9	7	9	7	9	7	9	7	5	7
STVH	0.396		0.814		0.41		0.18		0.671	
	2	1	2	3	3	2	4	2	4	4
	6	9	6	7	5	8	4	8	4	6
MIAMI	NA		NA		NA		NA		0.14	
	0	0	0	0	0	0	0	0	2	0
	12	12	12	12	12	12	12	12	10	12

Results of Bio-Statistician's Chi-Squared Analysis

Joint Tenderness: (0 - 3 Scale)

	30		60		120		180		1 wk	
Overall	0.004		0.088		0.216		0.101		0.565	
	19	7	17	10	17	12	20	13	15	13
	29	43	31	40	31	38	28	37	33	37
MSN-VA	0.018		0.047		0.51		0.51		0.115	
	4	0	3	0	3	2	3	2	2	0
	5	10	6	10	6	8	6	8	7	10
NEBR	NA		NA		NA		NA		0.531	
	0	0	0	0	0	0	0	0	1	2
	10	10	10	10	10	10	10	10	9	8
RAC-NYC	0.6		0.6		0.6		0.072		0.312	
	2	1	2	1	2	1	3	0	3	1
	7	7	7	7	7	7	6	8	6	7
STVH	0.04		0.019		0.094		0.019		0.914	
	7	4	8	5	7	5	8	5	3	4
	1	6	0	5	1	5	0	5	5	6
MIAMI	0.083		1		0.673		1		1	
	6	2	4	4	5	4	6	6	6	6
	6	10	8	8	7	8	6	6	6	6

Results of Bio-Statistician's Chi-Squared Analysis

	Fifty-Foot Walking Time (secs)											
	30		60		120		180		1 wk			
Overall	0.07		0.086		0.394		0.494		0.376			
	27	19	34	27	30	27	32	30	32	29		
	21	31	14	23	18	23	16	20	16	21		
MSN-VA	0.252		0.809		0.809		0.515		0.515			
	4	2	5	5	5	5	4	3	4	3		
	5	8	4	5	4	5	5	7	5	7		
NEBR	0.16		0.178		0.178		0.639		0.371			
	5	2	7	4	6	3	7	6	6	4		
	5	8	3	6	4	7	3	4	4	6		
RAC-NYC	0.819		0.232		0.49		0.2		0.453			
	4	4	7	4	7	5	8	5	8	6		
	5	4	2	4	2	3	1	3	1	2		
STVH	0.196		0.814		0.814		0.814		0.18			
	7	6	6	7	6	7	6	7	4	8		
	1	4	2	3	2	3	2	3	4	2		
MIAMI	0.414		0.386		0.682		0.386		0.346			
	7	5	9	7	6	7	7	9	10	8		
	5	7	3	5	6	5	5	3	2	4		

Results of Bio-Statistician's Chi-Squared Analysis

Global Evaluation - Patient: (0 - 10)

	30		60		120		180		1 wk	
Overall	0.312		0.161		0.162		0.161		0.106	
	17	13	23	17	24	18	25	19	28	21
	31	37	25	33	24	32	23	31	20	29
MSN-VA	0.153		0.463		0.21		0.405		0.405	
	1	4	3	5	2	5	2	4	2	4
	8	6	6	5	7	5	7	6	7	6
NEBR	NA		1		1		0.531		0.653	
	0	0	1	1	1	1	1	2	6	5
	10	10	9	9	9	9	9	8	4	5
RAC-NYC	0.149		0.064		0.086		0.086		0.232	
	4	1	5	1	6	2	6	2	7	4
	5	7	4	7	3	6	3	6	2	4
STVH	0.168		0.138		0.138		0.04		0.41	
	5	3	6	4	6	4	7	4	3	2
	3	7	2	6	2	6	1	6	5	8
MIAMI	0.414		0.408		0.206		0.386		0.083	
	7	5	8	6	9	6	9	7	10	6
	5	7	4	6	3	6	3	5	2	6

Results of Bio-Statistician's Chi-Squared Analysis

Global Evaluation - M.D.: (0 - 10)

	30		60		120		180		1 wk	
Overall	0.062		0.108		0.027		0.044		0.026	
	20	12	24	17	27	17	28	19	29	19
	28	38	24	33	21	33	20	31	19	31
MSN--A	0.313		0.21		0.21		0.405		0.405	
	1	3	2	5	2	5	2	4	2	4
	8	7	7	5	7	5	7	6	7	6
NESF	NA		0.305		0.305		0.531		0.371	
	0	0	1	0	1	0	2	1	6	4
	10	10	9	10	9	10	8	9	4	6
RAL-R:C	0.064		0.201		0.03		0.03		0.03	
	5	1	5	2	7	2	7	2	7	2
	4	7	4	6	2	6	2	6	2	6
STVH	0.015		0.007		0.007		0.019		0.737	
	7	3	8	4	8	4	8	5	3	3
	1	7	0	6	0	6	0	5	5	7
MIAMI	0.414		0.408		0.206		0.386		0.038	
	7	5	8	6	9	6	9	7	11	6
	5	7	4	6	3	6	3	5	1	6

Results of Biostatistician's Chi-Squared Analysis

	Pain Log Each Day (1 - 10 Scale)											
	2		3		4		5		6			
Overall	0.91		0.023		0.147		0.216		0.051			
	12	13	22	12	18	12	17	12	17	9		
	36	37	26	38	30	38	31	38	31	41		
MSN-VA	0.466		0.018		0.115		0.213		0.047			
	2	1	4	0	2	0	3	1	3	0		
	7	9	5	10	7	10	6	9	6	10		
NEBR	0.653		0.639		0.639		1		0.639			
	4	5	7	6	6	7	7	7	7	6		
	6	5	3	4	4	3	3	3	3	4		
RAC-NYC	0.312		0.064		0.312		0.156		0.312			
	3	1	5	1	3	1	2	0	3	1		
	6	7	4	7	6	7	7	8	6	7		
STAP	0.18		0.375		0.867		0.396		0.867			
	0	2	1	3	1	1	2	1	1	1		
	8	8	7	7	7	9	6	9	7	9		
MIAMI	0.653		0.178		0.206		1		0.273			
	3	4	5	2	6	3	3	3	3	1		
	9	8	7	10	6	9	9	9	9	11		

Appendix A

The Chi-Squared Technique

where

$$\{yx_k\} = \sum y_i x_{ki} - \left(\sum y_i\right)\left(\sum x_{ki}\right)\Big/ n$$

$$\{y^2\} = \sum y_i^2 - \left(\sum y_i\right)^2 \Big/ n$$

This analysis is for linear, noninteracting, independent variables; however, the analysis can be extended to include cases where the regression equations would have higher-order and cross-product terms. The nonlinear terms can enter the regression equation in an additive manner and are treated as an extra variable. With well-established computer routines for regression analysis, the set of $(k + 1)$ simultaneous equations given by Eqs. (10.37) can be solved quickly and inexpensively and no difficulties are encountered in adding extra terms to account for nonlinearities and interactions.

10.8 CHI-SQUARE TESTING

Once a series of measurements is made to obtain data to statistically characterize a population, it is possible to select a distribution function for the population and to predict population properties, such as the probability of occurrence of a certain event. This is an extremely important measurement procedure in engineering; therefore, to avoid error, it is essential that the correct distribution function be selected to represent the population. The data from a series of measurements can be subjected to a chi-square (χ^2) test to check the validity of the assumed distribution function.

The χ^2 statistic is defined as

$$\chi^2 = \sum \frac{(O - E)^2}{E} \tag{10.39}$$

where O is an observed number.
E is an expected number, based on a specified statistical distribution function.

The value χ^2 is used to determine how well the data fit the assumed statistical distribution. If $\chi^2 = 0$, the match is perfect. Values of $\chi^2 > 0$ indicate a possibility that the data are not represented by the specified distribution function. The probability p of χ^2 occurring as a result of random variation is listed in Table 10.7 and illustrated in Fig. 10.9. The number of degrees of freedom associated with any sequence of measurements is given by the expression

$$d = n - k \tag{10.40}$$

TABLE 10.7 Chi-Squared (χ^2) Values with Different Degrees of Freedom for Different Probability Levels

Degrees of Freedom	Probability (%)													
	1.0	2.5	5.0	10.0	20.0	30.0	40.0	50.0	60.0	70.0	80.0	90.0	95.0	99.0
1	.000157	.000982	.00393	.0158	.0642	.148	.275	.455	.708	1.07	1.64	2.71	3.84	6.63
2	.0201	.0506	.103	.211	.446	.713	1.02	1.39	1.83	2.41	3.22	4.61	5.99	9.21
3	.115	.216	.352	.584	1.00	1.42	1.87	2.37	2.95	3.67	4.64	6.25	7.81	11.3
4	.297	.484	.711	1.06	1.65	2.19	2.75	3.36	4.04	4.88	5.99	7.78	9.49	13.3
5	.554	.831	1.15	1.61	2.34	3.00	3.66	4.35	5.13	6.06	7.29	9.24	11.1	15.1
6	.872	1.24	1.64	2.20	3.07	3.83	4.57	5.35	6.21	7.23	8.56	10.6	12.6	16.8
7	1.24	1.69	2.17	2.83	3.82	4.67	5.49	6.35	7.28	8.38	9.80	12.0	14.1	18.5
8	1.65	2.18	2.73	3.49	4.59	5.53	6.42	7.34	8.35	9.52	11.0	13.4	15.5	20.1
9	2.09	2.70	3.33	4.17	5.38	6.39	7.36	8.34	9.41	10.7	12.2	14.7	16.9	21.7
10	2.56	3.25	3.94	4.87	6.18	7.27	8.30	9.34	10.5	11.8	13.4	16.0	18.3	23.2
11	3.05	3.82	4.57	5.58	6.99	8.15	9.24	10.3	11.5	12.9	14.6	17.3	19.7	24.7
12	3.57	4.40	5.23	6.30	7.81	9.03	10.2	11.3	12.6	14.0	15.8	18.5	21.0	26.2
13	4.11	5.01	5.89	7.04	8.63	9.93	11.1	12.3	13.6	15.1	17.0	19.8	22.4	27.7
14	4.66	5.63	6.57	7.79	9.47	10.8	12.1	13.3	14.7	16.2	18.2	21.1	23.7	29.1
15	5.23	6.26	7.26	8.55	10.3	11.7	13.0	14.3	15.7	17.3	19.3	22.3	25.0	30.6
20	8.26	9.59	10.9	12.4	14.6	16.3	17.8	19.3	21.0	22.8	25.0	28.4	31.4	37.6
25	11.5	13.1	14.6	16.5	18.9	20.9	22.6	24.3	26.1	28.2	30.7	34.4	37.7	44.3
30	15.0	16.8	18.5	20.6	23.4	25.5	27.4	29.3	31.3	33.5	36.3	40.3	43.8	50.9
40	22.2	24.4	26.5	29.1	32.3	34.9	37.1	39.3	41.6	44.2	47.3	51.8	55.8	63.7
50	29.7	32.4	34.8	37.7	41.4	44.3	46.9	49.3	51.9	54.7	58.2	63.2	67.5	76.2
100	70.1	74.2	77.9	82.4	87.9	92.1	95.8	99.3	102.9	106.9	111.7	118.5	124.3	135.8

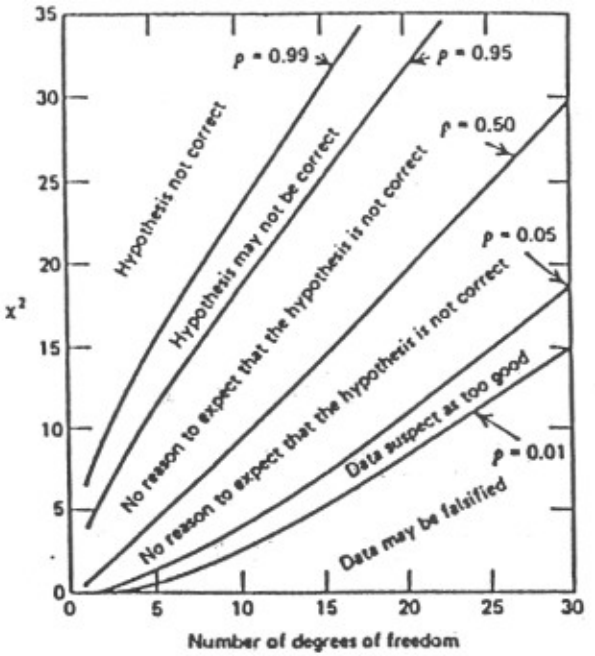


Figure 10.9 χ^2 values as a function of the number of degrees of freedom for different probability levels.

where n is the number of observations.
k is the number of conditions imposed on the distribution.

As an example, consider the yield strength data presented in Table 10.1. A χ^2 test can be used to determine how well these data are represented by a normal distribution having a mean $\bar{x} = 78.4$ ksi and a standard deviation $S_x = 6.04$ ksi. By using the properties of a normal distribution function, the number of specimens expected to fall in any strength group can be computed. The observed number of specimens in Table 10.1 exhibiting yield strengths within each of five group intervals, together with the computed number of specimens in a normal distribution in the same group intervals, are listed in Table 10.8. Also, the computation of the χ^2 value ($\chi^2 = 1.129$) is illustrated in the table. Since the number of groups is 5 ($n = 5$) and since the two distribution parameters $\bar{x}$ and S_x were determined by using these data (thus, $k = 2$), the number of degrees of freedom is $d = n - k = 5 - 2 = 3$. Thus, from Table 10.7, it can be concluded with a 77 percent probability that the data of Table 10.1 can be represented by a normal distribution function. This can be considered a very good fit, as shown in Fig. 10.9. One could conclude in other instances with higher χ^2 values and much lower probability levels that the assumed distribution was representative.

TABLE 10.8 χ^2 Computation for Grouped Yield Strength Data

Group Interval	Number Observed	Number* Expected	$(O - E)^2/E$
0 –69.99	2	1.646	0.0761
70.0–74.99	3	4.108	0.2988
75.0–79.99	8	6.298	0.4600
80.0–84.99	4	5.190	0.2729
85.0–∞	3	2.758	0.0212
Total	20	20	1.129 $= \chi^2$

* Based on a normal (Gaussian) distribution of data.

The χ^2 statistic can also be used in contingency testing where the sample is classified under one of two catagories—say pass or fail. Consider, for example, an inspection procedure with a particular type of strain gage where 10 percent of the gages are rejected due to etching imperfections in the grid. In an effort to reduce this rejection rate, the manufacturer has introduced new clean-room techniques that are expected to improve the quality of the grids. On the first lot of 1000 gages, the failure rate was reduced to 9 percent. Is this reduced failure rate due to chance variation, or have the new clean-room techniques improved the manufacturing process? A χ^2 test can establish the probability of the improvement being the result of random variation. The computation of χ^2 for this example is illustrated in Table 10.9.

The data in Table 10.7 with $d = 1$ give a probability p of χ^2 exceeding 1.111 of 29 percent; thus, the test provides a strong indication that the clean-room improvements have reduced the rejection rate. Stronger statistical statements could be made after testing of a second lot of 1000 gages is completed, if the trend continues. At this point 2000 gages would have been inspected with a value $\chi^2 = 2.222$. The probability of χ^2 exceeding this value due to random variation is only 14 percent.

TABLE 10.9 Observed and Expected Inspection Results

	Observed Number	Expected Number	$(O - E)^2/E$
Passed	910	900	0.111
Failed	90	100	1.000
			1.111 $= \chi^2$